Sequence and Series-

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9.4

Q3→

$$S_n = 3\cdot1^2 + 5\cdot2^2 + 7\cdot3^2 \ldots\ldots + (2n+1)n^2$$

$S_1 = 3, 5, 7, \ldots$

$a_n^1 = a+(n-1)d$

$3+(n-1)2$

$S_2 = 1^2, 2^2, 3^2 \ldots$

$a_n^2 = n^2$

$$a_n = (2n+1)n^2$$

$$S_n = \sum_{r=1}^{n} a_r = \sum_{r=1}^{n} (2r+1)r^2$$

$$= \sum_{r=1}^{n}(2r^3 + r^2)$$

$$= 2\sum_{r=1}^{n} r^3 + \sum_{r=1}^{n} r^2$$

$$= 2\left(\left[\frac{n(n+1)}{2}\right]^2\right) + \left(\frac{n(n+1)(2n+1)}{6}\right)$$

$$= \frac{n^2(n+1)^2}{2} + \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n(n+1)}{2}\left\{n(n+1) + \frac{2n+1}{3}\right\}$$

$$\Rightarrow \frac{n(n+1)}{2}\left\{\frac{3n^2+3n+2n+1}{3}\right\}$$

$$= \frac{n(n+1)}{2}\left\{\frac{3n^2+5n+1}{3}\right\}$$

$$\boxed{S_n = \frac{n(n+1)(3n^2+5n+1)}{6}}$$

Q4 → $S_n = \frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \cdots\cdots + \frac{1}{n(n+1)}$

$S_n = \frac{2-1}{1\times 2} + \frac{3-2}{2\times 3} + \frac{4-3}{3\times 4} + \cdots + \frac{(n+1)-n}{n(n+1)}$

$S_n \Rightarrow 1 - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}} + \cancel{\frac{1}{3}} - \cancel{\frac{1}{4}} + \cdots + \cancel{\frac{1}{n}} - \frac{1}{n+1}$

$S_n \Rightarrow 1 - \frac{1}{n+1} \Rightarrow \boxed{\frac{n}{n+1}}$

$a_n = \frac{1}{n(n+1)}$

Q7 → $S_n = 1^2 + (1^2+2^2) + (1^2+2^2+3^2) + \cdots\cdots + (1^2+2^2+3^2+\cdots n^2)$

$a_n = 1^2+2^2+3^2+\cdots+n^2$

$a_n = \frac{n(n+1)(2n+1)}{6} = \frac{n(2n^2+3n+1)}{6}$

$\boxed{a_n \Rightarrow \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}}$

$S_n = \sum_{r=1}^{n} a_r$

$\Rightarrow \sum_{r=1}^{n}\left(\frac{r^3}{3} + \frac{r^2}{2} + \frac{r}{6}\right)$

$\Rightarrow \frac{1}{3}\sum_{r=1}^{n} r^3 + \frac{1}{2}\sum_{r=1}^{n} r^2 + \frac{1}{6}\sum_{r=1}^{n} r$

$\Rightarrow \frac{1}{3}\left[\frac{n(n+1)}{2}\right]^2 + \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6}\right] + \frac{1}{6}\left[\frac{n(n+1)}{2}\right]$

$\Rightarrow \frac{n(n+1)}{12}\left[n(n+1) + (2n+1) + 1\right]$

$\Rightarrow \frac{n(n+1)}{12}\left[n^2 + n + 2n + 2\right]$

$n = 2$

$S_2 = \frac{2(3)(12)}{12} = 6$

$S_n \Rightarrow n(n+1)(n^2+3n+2)$

$S_n \Rightarrow \dfrac{n(n+1)(n^2+3n+2)}{12}$

$S_2 = \dfrac{1^2 + (1^2+2^2)}{1} \Rightarrow 6$

Mis. Examples →

$a \,\&\, d \leftarrow AP$

Q21. $p^{th}, q^{th}, r^{th} \,\&\, s^{th}$ term of an AP are in GP.

$a_p = a+(p-1)d$

$a_q = a+(q-1)d$

$a_r = a+(r-1)d$

$a_s = a+(s-1)d$

$\Downarrow$

GP

a_p, a_q, a_r, a_s

AP

2, 4, 6, 8, 10, 12, 14, 16

GP

Show $p-q,\ q-r,\ r-s$ are in GP

To prove

$$\frac{q-r}{p-q} = \frac{r-s}{q-r}$$

$$\frac{a_q}{a_p} = \frac{a_r}{a_q} = \frac{a_s}{a_r}$$

$$\frac{a+(q-1)d}{a+(p-1)d} = \frac{a+(r-1)d}{a+(q-1)d} = \frac{a+(s-1)d}{a+(r-1)d}$$

Property

$$\frac{a}{b} = \frac{c}{d}$$

CeD

① $\dfrac{a-b}{a+b} = \dfrac{c-d}{c+d}$

② $\dfrac{a-c}{b-d}$

$\dfrac{c}{d} = \dfrac{a}{b} = k \Rightarrow a = bk,\ c = dk$??

$\dfrac{a-c}{b-d} = \dfrac{bk-dk}{b-d} = k$

$$\frac{a_q}{a_p} = \frac{a_r}{a_q} = \frac{a_s}{a_r} = \frac{a_q-a_r}{a_p-a_q} = \frac{a_r-a_s}{a_q-a_r}$$

$\Downarrow$

$$\frac{\{a+(q-1)d\}-\{a+(r-1)d\}}{\{a+(p-1)d\}-\{a+(q-1)d\}} = \frac{\{a+(r-1)d\}-\{a+(s-1)d\}}{\{a+(q-1)d\}-\{a+(r-1)d\}}$$

$$\frac{(q-r)\cancel{d}}{(p-q)\cancel{d}} = \frac{(r-s)\cancel{d}}{(q-r)\cancel{d}}$$

$$\Rightarrow \quad (q-r)^2 = (p-q)(r-s) \qquad (b^2 = ac)$$

$(p-q), (q-r), (r-s)$ are in GP

Ex Q22 → a, b, c are in GP

$$a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}}$$

Show $x, y,$ and z are in AP

$$r = \frac{b}{a} = \frac{c}{b} \Rightarrow \boxed{b^2 = ac} \quad \text{—(1)}$$

Let $a^{\frac{1}{x}} = b^{\frac{1}{y}} = c^{\frac{1}{z}} = k$

$$\Downarrow$$

$$\boxed{a = k^x, \; b = k^y, \; c = k^z} \quad \text{—(2)}$$

Put (2) in (1)

$$(k^y)^2 = (k^x)(k^z)$$

$$k^{2y} = k^{x+z}$$

$$2y = x+z$$

$$y = \frac{x+z}{2} \Rightarrow \underline{x, y, z \text{ are in AP}}$$

$$d = \underline{y-x} = \underline{z-y}$$

—x—x—x—x—

Ex 23 →

$$\underset{+ve}{(a^2+b^2+c^2)}\, p^2 - 2(ab+bc+cd)p + (b^2+c^2+d^2) \le 0$$

a, b, c, d & p are different real numbers.

$$Ap^2 + Bp + C \le 0$$
$$\& \quad A > 0$$

$$a^2p^2 + b^2p^2 + c^2p^2 - 2abp - 2bcp - 2cdp + b^2 + c^2 + d^2 \le 0$$

$$(ap-b)^2 + (bp-c)^2 + (cp-d)^2 \le 0$$

This equation cannot be less than zero
It can be equal to zero only if all three terms are zero.

$ap - b = 0$, $bp - c = 0$, $cp - d = 0$

$p = \frac{b}{a}$ (1) $\quad p = \frac{c}{b}$ (2) $\quad p = \frac{d}{c}$ (3)

$\Rightarrow p = \frac{b}{a} = \frac{c}{b}, \frac{d}{c}$

a, b, c & d are in GP & p is common ratio.

— x — x — x —

Qx24 →

p, q & r are in GP

$\boxed{q^2 = pr}$ — (1)

$\boxed{p x^2 + 2qx + r = 0}$ — (2)

$dx^2 + 2ex + f = 0$ — (3)

have a common root

↓ show

$\frac{d}{p}, \frac{e}{q}, \frac{f}{r}$ are in AP

$\boxed{2\frac{e}{q} = \frac{d}{p} + \frac{f}{r}}$

Solving (2)

$$x = \frac{-2q \pm \sqrt{4q^2 - 4pr}}{2p}$$

$$x = \frac{-\cancel{2}q \pm \cancel{2}\sqrt{q^2 - pr}}{\cancel{2}p}$$

$(q^2 = pr)$ from (1)

$D = 0$

$\boxed{x = -\frac{q}{p}}$ (Repeating roots)

Only one root

this is the common root

Put $x = -\frac{q}{p}$ in (3) $\quad dx^2 + 2ex + f = 0$

$$d\left(\frac{-q}{p}\right)^2 + 2e\left(\frac{-q}{p}\right) + f = 0$$

$q^2 = pr$ (1)

$$\boxed{dq^2 - 2epq + fp^2 = 0} \quad \times p^2 \quad (4)$$

$$2epq = dq^2 + fp^2$$

divide by $p^2 r$

$$2e\cancel{p}q = d\,q^2 + f\,\cancel{p^2}$$

$$\frac{2e\cancel{p}q}{p^{\cancel{2}}r} = \frac{dq^2}{p^2 r} + \frac{f\cancel{p^2}}{\cancel{p^2}r}$$ divide by $p^2 r$

from (1)

$$\frac{2e\cancel{q}}{q^{\cancel{2}}} = \frac{d\,\cancel{pr}}{p^{\cancel{2}}\cancel{r}} + \frac{f}{r}$$

$$\frac{2e}{q} = \frac{d}{p} + \frac{f}{r}$$

$\frac{d}{p}, \frac{e}{q}, \frac{f}{r}$ are in AP

— x — x — x — x —

Mis ch-9

Q1- $S_{m+n} + S_{m-n} = 2S_m$ $\quad 2\left(\frac{m}{2}(2a+(m-1)d)\right)$

AP: a, d

LHS $= S_{m+n} + S_{m-n}$ $\quad S_n = \frac{n}{2}(2a+(n-1)d)$

$$\frac{(m+n)}{2}\left(2a+(m+n-1)d\right) + \left(\frac{m-n}{2}\right)\left(2a+(m-n-1)d\right)$$

$$(m+n)a + \frac{(m+n)(m+n-1)d}{2} + (m-n)a + \frac{(m-n)(m-n-1)d}{2}$$

$$2ma + \frac{d}{2}\left\{m^2+\cancel{mn}-m+\cancel{nm}+n^2-\cancel{n}+m^2-\cancel{mn}-m-\cancel{nm}+n^2+\cancel{n}\right\}$$

$$2ma + \frac{d}{2}\left\{2m^2+2n^2-2m\right.$$

Q1-

LHS $\quad a_{m+n} + a_{m-n}$ $\quad$ — x — x — x —

$$a+(m+n-1)d + a+(m-n-1)d$$

$$2a + 2md - 2d$$

$$2\left(a+(m-1)d\right) = 2a_m$$

Q2→ Let 3 numbers be . . .

$$a-d,\ a,\ a+d$$

Sum

$$(a-d)+a+(a+d)=24$$

$$3a=24$$

$$\boxed{a=8} \quad —(1)$$

Product

$$(a-d)(a)(a+d)=440$$

$$a(a^2-d^2)=440$$

$$\cancel{8}(64-d^2)=\cancel{440}\,55$$

$$d^2=9 \qquad \boxed{d=\pm 3} \quad (2)$$

From (1) & (2)

3 number are $\overline{\underline{5, 8, 11}}$

—x—x—x—

Q3→

$$S_3=3(S_2-S_1)$$

$S_1 \Rightarrow$ sum of n terms

$$\boxed{S_1=\frac{n}{2}\left(2a+(n-1)d\right)}$$

$S_2=$ Sum of 2n terms

$$\boxed{S_2=\frac{2n}{2}\left(2a+(2n-1)d\right)}$$

$$\boxed{S_3=\frac{3n}{2}\left(2a+(3n-1)d\right)}$$

—x—x—x—

$$RHS=3\left(S_2-S_1\right)$$

$$=3\left(\frac{\cancel{2}n}{\cancel{2}}\left(2a+(2n-1)d\right)-\frac{n}{2}\left(2a+(n-1)d\right)\right)$$

$$=3\left\{2na+(2n^2-n)d-na-\frac{(n^2-n)}{2}d\right\}$$

$$\Rightarrow 3\left\{na+\left(2n^2-n-\frac{n^2}{2}+\frac{n}{2}\right)d\right\}$$

$$=3\left\{na+\left(\frac{3}{2}n^2-\frac{n}{2}\right)d\right\}$$

$$=3n\left\{\frac{2a}{2}+\left(\frac{3n}{2}-\frac{1}{2}\right)d\right\}$$

$$\Rightarrow \frac{3n}{2}\left(2a+(3n-1)d\right) \Rightarrow \underline{\overline{S_3}}=\underline{\overline{LHS}}$$

Q4 → 200 & 400 which are divisible by 7

Dividing 200 by 7 ⇒ remainder is 4

↓ Add 3

203 is divisible by 7

400 by 7

↓ −1

399

$$7\overline{)400}$$ 57, 35, 50, 49, 1

$$S = 203 + 210 + 217 + \dots + 399$$

$a = 203, d = 7, \quad a_n = 399$

$$a_n = a + (n-1)d$$

$$399 = 203 + (n-1)7$$

$$196 = (n-1)7$$

$$28 = (n-1) \qquad \boxed{n = 29}$$

$$S = \frac{n}{2}(a + a_n)$$

$$\Rightarrow \frac{29}{2}(203 + 399)$$

$$= \frac{29 \times \cancel{602}^{301}}{\cancel{2}}$$

$$\Rightarrow \boxed{8729}$$

— x — x — x —

— x — x — x — x —

Q5 → 1 to 100 divisible by 2 or 5

divisible by 2: 2, 4, 6, 8, 10, 12, ... 100

+

divisible by 5: 5, 10, 15, ..., 100

−

divisible by 2 & 5: 10, 20, ... + 100

$a = 2$
$d = 2$

$$100 = 2 + (n-1)2$$
$$n = 50$$

$a = 5$
$d = 5$

$$100 = 2 + (n-1)d$$
$$n = 20$$

$a = 10$
$d = 100$

$$100 = 10 + (n-1)10$$
$$n = 10$$

$$\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$$

OR   AND

~~2+4+6+8+10+12+14+15~~

$$S_{2015} = S_2 + S_5 - S_{2\&5}$$

$$\Rightarrow \frac{\cancel{50}}{\cancel{2}}\left(2(2)+(50-1)2\right) + \frac{\cancel{20}^{10}}{\cancel{2}}\left(2(5)+(20-1)5\right)$$

$$- \frac{10}{\cancel{2}}\left(2(10)+(10-1)\cancel{10}^{5}\right)$$

$$\Rightarrow 50\times 51 + 10\times 105 - 10(55)$$

$$\Rightarrow 2550 + 1050 - 550$$

$$\Rightarrow 2000 + 1050 \Rightarrow \boxed{\cancel{3050}}$$

$$= \boxed{3050}$$

— x — x — x — x —

Q7 →

$$\boxed{f(x+y) = f(x)f(y)} \quad \text{— (1)}$$

$$\boxed{f(1) = 3} \quad \text{— (2)}$$

$$\sum_{x=1}^{n} f(x) = 120$$

$$\boxed{f(1)+f(2)+f(3)+\dots+f(n) = 120} \qquad n = ??$$

Ex...

$$a^{x+y} = a^x a^y$$

$$f(x) = a^x$$

~~from (1) & (2)~~

put $x=1$ & $y=1$ in (1)

$$f(1+1) = f(1)f(1)$$

from (2)

$$f(2) = 3\times 3$$

$$f(2) = 9$$

$$3 + 9 + 27 \dots = 120$$

$a = 3$

$r = 3$

$$\frac{a(r^n-1)}{(r-1)} = S_n$$

$$f(3) = f(2+1)$$

$$\Rightarrow f(2)f(1)$$

$$\Rightarrow 9\times 3$$

$$= 27\dots$$

~~— x — x — x — x —~~

$$\frac{\cancel{3}(3^n-1)}{(3-1)} = \cancel{120}\ 40$$

$$3^n - 1 = 80 \Rightarrow 3^n = 81$$

$$3^n = 3^4$$

$$\Rightarrow \boxed{n = 4}$$

— x — x — x — x — x — x —

Q18 →

$$x^2 - 3x + p = 0 \Rightarrow \text{Roots } a \,\&\, b$$

$$\boxed{a+b = 3} \text{ —(1)} \qquad \boxed{ab = p} \text{ —(1)}$$

Sum of Roots

$$\alpha + \beta = \frac{-b}{a}$$

Product

$\cancel{a+b}$ $\boxed{a+b=3}$ — ① $\boxed{ab=p}$ — ①

$x^2-12x+q=0$ → Roots c & d

$\boxed{c+d=12}$ — ② $\boxed{cd=q}$ — ②

$\alpha+\beta=\frac{-b}{a}$

Product of Roots $\alpha\beta=\frac{c}{a}$

Prove $\boxed{\frac{q+p}{q-p}=\frac{17}{15}}$ $\Rightarrow$ $15q+15p=17q-17p$

$32p=2q$

$\boxed{\frac{p}{q}=\frac{1}{16}}$

a,b,c,d are in GP

$\frac{b}{a}=\frac{c}{b}=\frac{d}{c}=\cancel{r}$ ← common ratio

a, ar, ar^2, ar^3
$\| \quad \| \quad \| \quad \|$
$a \quad b \quad c \quad d$

from ①

$a+b=3$

$a+ar=3$

$\boxed{a(1+r)=3}$

from ②

$c+d=12$

$ar^2+ar^3=12$

$r^2 a(1+r)=12$

$r^2(3)=12$

$r^2=4$

$\boxed{r=\pm 2}$

Case 1

$r=+2$

$a(1+2)=3$

$\boxed{a=1}$ $\boxed{r=2}$

$1,2,4,8$

$\Rightarrow \frac{q+p}{q-p}=\frac{cd+ab}{cd-ab}=\frac{(ar^2)(ar^3)+a(ar)}{(ar^2)(ar^3)-a(ar)}$

$=\frac{\cancel{a^2r}(r^4+1)}{\cancel{a^2r}(r^4-1)}\Rightarrow\frac{16+1}{16-1}=\boxed{\frac{17}{15}}$

Case 2

$(-2))=3$

$a(-1)=3$

$\boxed{a=-3}$ $\boxed{r=-2}$

$-3, 6, -12, 24$

$\Rightarrow \frac{q+p}{q-p}=\frac{r^4+1}{r^4-1}=\frac{16+1}{16-1}=\boxed{\frac{17}{15}}$

Q17 → a, b, c & d are in GP

a, ar, ar^2, ar^3

Prove

$(a^n+b^n), (b^n+c^n), (c^n+d^n)$ are in GP

$$\frac{a_2}{a_1} = \frac{b^n+c^n}{a^n+b^n} = \frac{(ar)^n+(ar^2)^n}{a^n+(ar)^n} \Rightarrow \frac{\cancel{a^n}r^n\cancel{(1+r^n)}}{\cancel{a^n}\cancel{(1+r^n)}} = r^n$$

$$\frac{c^n+d^n}{b^n+c^n} = \frac{(ar^2)^n+(ar^3)^n}{(ar)^n+(ar^2)^n} = \frac{\cancel{a^n}r^{2n}\cancel{(1+r^n)}}{\cancel{a^n}r^n\cancel{(1+r^n)}} = r^n$$

Hence proved